718. Give a simple expression for $\sin x$, where x, expressed in degrees, is a positive integer divisible by 1000, say

$$x = \sum_{k=3}^{n} a_k 10^k \quad (0 \leqslant a_k \leqslant 9).$$

(S. RIENSTRA)

Solutions by A.A. JAGERS, R.A. KORTRAM, L. KUIPERS, S. RIENSTRA (2 solutions).

SOLUTION by R.A. KORTRAM.
Because of $1000 - 3 \times 360 = -80$ we see that $10^3 \equiv -80 \pmod{360}$. By induction, it follows that $10^k \equiv -80 \pmod{360}$ for $k \geqslant 3$. Hence

$$x = \sum_{k=3}^{n} a_k 10^k \equiv -80 \sum_{k=3}^{n} a_k \pmod{360}.$$

Consequently,

$$\sin x = -\sin(80 \sum_{k=3}^{n} a_k)$$